Triangulated Categories

1. Fill in the details of some of the proofs I gave in class showing that the homotopy category of chain complexes is triangulated.

2. Let $\mathcal{C}$ be a triangulated category, with translation functor T. and let

$$A \xrightarrow{u} B \xrightarrow{v} C \xrightarrow{w} T(A)$$

be a distinguished triangle. Prove the following statements.

   (a) uv and wv are zero.

   (b) If X is any object of $\mathcal{C}$ the sequences

$$\mathrm{Hom}(X, A) \longrightarrow \mathrm{Hom}(X, B) \longrightarrow \mathrm{Hom}(X, C) \longrightarrow \mathrm{Hom}(X, T(A))$$

$$\mathrm{Hom}(T(A), X) \longrightarrow \mathrm{Hom}(C, X) \longrightarrow \mathrm{Hom}(B, X) \longrightarrow \mathrm{Hom}(A, X)$$

   are exact.

   (c) In a morphism of distinguished triangles, if any two of the maps are isomorphisms, so is the third. (Hint: use the previous exercise). Note that this implies that a distinguished triangle containing a morphism u is unique up to nonunique isomorphism.